DESCRIPTION LOGICS

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1 INTRODUCTION

One of the key objectives of research in Artificial Intelligence (AI) from its very beginning is the ability to represent information on a domain of interest in a compact way and, at the same time, to derive implicit information from this representation. This field in AI research is known as Knowledge Representation and Reasoning (KRR).

Description Logics (DLs) [Baader *et al.*, 2007a; Baader *et al.*, 2007b; Calvanese *et al.*, 2001; Hitzler *et al.*, 2010] emerged within KRR research from early network-based approaches, by building on their structured/taxonomic organization of a terminology of a domain of interest, but equipping it with a well-understood logic-based semantics. In fact, concepts (classes of objects) can be interpreted as unary predicates and roles (properties linking such classes) as binary predicates, and complex expressions can be built using logic-based constructors in an inductive way.

This formal underpinning proves very useful when developing inference services in Description Logics, all the more so, as it turns out that the full expressive power of first-order logic is often not required. Rather, a decidable fragment of it usually suffices, which paves the way for efficient reasoning procedures tailored to the particular language, i.e., to the necessary complex constructors, and to the concrete KRR application in mind.

Overall, the research in Description Logics and the development of KRR systems that build on DLs follow a number of important principles that distinguish the area from others in KRR research.

First, as already pointed out, the terms in the knowledge base are organized in a taxonomic, ontological way and the semantics based on first-order logics strictly adheres to the so-called open world assumption, in which inferences can only be drawn based on the content explicitly present in the knowledge base. This makes DLs particularly suitable for applications where these properties are beneficial or even required, such as modeling ontologies in the Semantic Web (cf. the corresponding chapter in this volume).

Second, as first introduced in [Brachman and Levesque, 1984], a crucial idea pursued in DL research is the trade-off between the expressive power of the language (given the provided complex constructors) and the computational complexity of reasoning in that language. In this sense, one has to balance the admitted complex constructors for an application domain w.r.t. what is expressible and how fast inferences can be obtained. One should note here, that the aim is always to obtain a balance in which reasoning tasks are at least decidable, (independently of the concrete computational complexity), unlike other KRR formalisms that may trade decidability for more expressive power.

A third principle is the close connection between theory and practice. This is witnessed throughout the history of DLs by the mutual direct impact theoretical research and implementations (driven by actual applications, such as in natural language processing, database management, medical informatics, and software engineering to name but a few) had on each other.

Illustrating these principles, in this chapter we give an overview of Description Logics as a KRR formalism, pointing out its historic roots (Sect. 2), its syntax and semantics (Sect. 3), and algorithmic aspects (Sect. 4). We also discuss recent developments in DLs (Sect. 5) before we conclude (Sect. 6). Note that the material presented here is not meant to be exhaustive. More details can be found in [Baader *et al.*, 2007a; Baader *et al.*, 2007b; Calvanese *et al.*, 2001; Hitzler *et al.*, 2010]. In particular the DL Handbook [Baader *et al.*, 2007a] offers a very detailed account on all aspects of theory, implementation, and application of Description Logics. Furthermore, the chapter *Logics for the Semantic Web* (in this volume) discusses Description Logics in the context of the Semantic Web field, which is its currently most prominent application area.

2 HISTORIC ROOTS

Knowledge Representation and Reasoning as a field became considerably popular in the early 1970s and a large variety of different approaches emerged, whose underlying motivations and rationales differed substantially. Among these proposals, logic-based formalisms stem from the idea that a formalization in first-order predicate logic is most suitable due to its generality and highly expressive language. That, however, also means that reasoning, i.e., the computation of logical deductions, is in general not decidable, and also usually considerably less efficient than reasoning in a language that is tailored to the requirements of the application in mind (see, e.g., [Tsarkov *et al.*, 2004] for a more recent such comparison).

At the same time, a variety of non-logic-based formalisms were introduced. Common to them is that they were obtained by observing human behavior on resolving certain tasks or by performing cognitive experiments, and then creating formal representations that model the observations and emulate intelligent behavior. Though based on concrete observations, it was then expected that such systems would be applicable in general also to other problem domains. Unlike the logic-based approaches, knowledge representation and reasoning was achieved in a rather ad-hoc manner driven by the specific needs of an application and therefore potentially quite different from one application to the other.

One such early formalism is rule-based expert systems, such as Mycin that, with around 450 IF-THEN rules, was able to diagnose blood infections in a similar manner as some experts and better than junior doctors (see [Russell and Norvig, 2010], also for other systems similar in spirit). These systems were, however, criticized for their lack of structure in the represented information (see, e.g., [Lehmann, 1992a]), in the sense that neither the system nor the non-expert human reader had any way to distinguish whether the encoded information was meaningful or not, so the entire search space had to be considered when reasoning, and consequently the re-use of expert systems for other purposes was rather difficult.

So-called network-based structures seemed to offer a solution for providing the (expert) knowledge in a more structured way. Network-based structures [Lehmann, 1992a; Lehmann, 1992b] themselves also represent a variety of approaches, the first being semantic networks [Quillian, 1967], in which a model of human memory is created by transferring information from a dictionary into a network of elements and their interconnections. Another prominent approach are frame systems [Minsky, 1981], in which frames serve as prototypes and relationships between such frames are expressible. Despite their differences, common to these early KRR formalisms is the objective to model sets of classes and the relations between these classes in a structured, taxonomic way.

However, the lack of formal semantics for these network-based structures meant that structured information could still be ambiguous and was therefore cause for considerable criticism [Brachman, 1977; Hayes, 1977; Hayes, 1979; Woods, 1975]. For example, a relation between two classes of individuals could mean that either there is some relation between the individuals of these classes, or that the relation is true for all individuals of these related classes, or even that the relation only holds (by default) as long as no knowledge to the contrary is explicitly available (see also [Brachman, 1983; Palomki and Kangassalo, 2012]). The consequence of this ambiguity is that many early systems building on network-based structures behave differently despite appearing to be almost identical.

In [Hayes, 1979], it was realized that a formal semantics could be provided for frames, basically by relying on first-order predicate logic: sets of individuals can be represented by unary predicates, and relations between such sets can be represented by binary predicates. One may wonder now whether this would not result simply in a logic-based formalism as described before including its disadvantages. As it turns out, network-based structures do not require the full expressiveness of first-order logic [Brachman and Levesque, 1985]. It suffices to use fragments of it, and the varying features in those network-based structures can then be represented by different (Boolean) constructors resulting in different fragments of first-order logic. As a consequence, it was recognized that reasoning in such structure-based representations could be achieved by specialized reasoners without relying on full first-order theorem provers. In addition to that, it was discovered that there is a trade-off between the expressive power of the language resulting from the inductive combination of the admitted language constructors and the computational properties of that language [Brachman and Levesque, 1984]. This also introduced the idea of studying computational properties in terms of computational complexity to the area of DLs, and mapping out the computational complexity of different DL languages became one of the driving forces in DL research.

The first system based on such a formal semantics was KL-ONE [Brachman and Schmolze, 1985], which is based on structured inheritance networks [Brachman, 1977; Brachman, 1978]. As pointed out in [Nardi and Brachman, 2007], KL-ONE introduced many of the key notions used in Description Logics, for example, the notions of concepts and roles and the relation between them; "value restriction" and "number restriction" as new ideas that changed the use of roles when defining concepts; and the reasoning tasks of subsumption and classification (see Sect. 3 for explanations on these terms). It also paved the way towards the later distinction between TBox and ABox and provided a first example of the close connection between theory and practice in Description Logics. Moreover, KL-ONE triggered the appearance of so-called hybrid systems, such as KRYPTON [Brachman *et al.*, 1983], that combined an expressive logic- or rule-based reasoner for the ABox with a taxonomic reasoner for the TBox, and the examination and evaluation of KL-ONE and similar systems then would be the starting point for description logic systems that will be briefly described in Sect. 4.[1]

3 SYNTAX AND SEMANTICS

Traditionally speaking, a description logic is a decidable fragment of first-order predicate logic,[2] where decidability is obtained by disallowing function symbols and by suitably restricting the use of quantifiers. We will formally introduce the description logic $\mathcal{ALC}$ (from *Attributive Logic with Complement*), which is usually considered to be the most *basic* description logic. We will also discuss some prominent extensions and fragments of $\mathcal{ALC}$. While this is only a very brief introduction, we refer the reader to [Baader *et al.*, 2007a; Baader *et al.*, 2007b; Hitzler *et al.*, 2010] for further details.

Let N_C, N_R, and N_I be countably infinite sets of *concept names*, *role names*, and *individual names*, respectively. Concept names are also called *atomic concepts* or *atomic classes. Complex concepts* (in short, *concepts*) can now be formed according to the grammar

$$C ::= \top \mid \bot \mid A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists R.C \mid \forall R.C,$$

where $A \in N_C$ is an atomic concept, $R \in N_R$ is a role, and C, D are complex concepts. $\top$ is called the *top concept*, while $\bot$ is called the *bottom concept.* A *general inclusion axiom* (*GCI*) is a statement of the form $C \sqsubseteq D$, where C and D are concepts. A *TBox* is a finite set of general inclusion axioms. An *ABox* is a

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[1] More details on these DL systems and its predecessors can be found in the chapter on Description Logic Systems in the DL Handbook [Möller and Haarslev, 2007].

[2] However, see Sect. 5 for pointers to recent developments, which sometimes incorporate alternative semantics.

$\top$	*true*
$\bot$	*false*
A	$A(x)$
$\neg C$	$\neg C(x)$
$C \sqcap D$	$C(x) \wedge D(x)$
$C \sqcup D$	$C(x) \vee D(x)$
$\exists R.C$	$\exists y(R(x,y) \wedge D(y))$
$\forall R.C$	$\forall y(R(x,y) \rightarrow D(y))$
$C \sqsubseteq D$	$\forall x(C(x) \rightarrow D(x))$
$C(a)$	$C(a)$
$R(a,b)$	$R(a,b)$

Table 1. Translating description logic axioms into first-order predicate logic.

finite set of *concept assertion axioms* and *role assertion axioms*. The former are of the form $C(a)$, where C is a concept and $a \in N_I$, and the latter are of the form $R(a,b)$, where $a, b \in N_I$ and $R \in N_R$. An $\mathcal{ALC}$ *knowledge base* is a union of an ABox and a TBox.

In terms of first-order predicate logic, individuals are constants, concepts are unary predicates, and roles are binary predicates. In fact, every axiom can be translated directly into first-order predicate logic as indicated in Table 1. Of course, this translation has to be applied recursively, with suitable variable renamings. $\mathcal{ALC}$ indeed inherits its model-theoretic semantics from first-order predicate logic by means of this translation.

With the semantics in place, a number of standard inference problems can be defined. Given an $\mathcal{ALC}$ knowledge base $\mathcal{K}$, $\mathcal{K}$ is called *consistent* if it has a model. A concept C is *satisfiable* w.r.t. $\mathcal{K}$ if there exists a model $\mathcal{I}$ of $\mathcal{K}$ with $C^{\mathcal{I}} \neq \emptyset$, in which case we call $\mathcal{I}$ a *model of* C w.r.t. $\mathcal{K}$. Concept C *is subsumed by* concept D w.r.t. $\mathcal{K}$, written $C \sqsubseteq_{\mathcal{K}} D$, if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{K}$. Two concepts, C and D are *equivalent* w.r.t. $\mathcal{K}$, written $C \equiv_{\mathcal{K}} D$, if C is subsumed by D and vice-versa, and *disjoint* w.r.t. $\mathcal{K}$ if $C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$ for every model $\mathcal{I}$ of $\mathcal{K}$. Also, an individual a is an *instance of a concept* C w.r.t. $\mathcal{K}$, written $\mathcal{K} \models C(a)$, if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{K}$. Likewise, a pair of individuals (a,b) is an *instance of a role* r w.r.t. $\mathcal{K}$, written $\mathcal{K} \models r(a,b)$, if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{K}$.

The definition of these reasoning tasks carries over to other DLs. Depending on the constructors available in a concrete DL, reasoning tasks can be reduced to each other, which means that quite often only one of those tasks has to be considered when developing inference engines. This applies for example to $\mathcal{ALC}$ [Baader and Nutt, 2007], and it was shown that reasoning in $\mathcal{ALC}$ is decidable [Schmidt-Schauß and Smolka, 1991], namely when establishing that concept satisfiability in $\mathcal{ALC}$ is PSpace-hard for acyclic TBoxes, i.e., where GCIs do not form cycles, and later also ExpTime-complete in general [Schild, 1991; Donini and Massacci, 2000]. More details on complexity and related algorithms follow in Sect. 4.

$\leq nR.C$	$\mathcal{Q}$	max card.	$\bigwedge_{i=1}^{n+1}(R(x,y_i) \wedge C(y_i)) \rightarrow \bigvee_{i,j} y_i = y_j$
$\geq nR.C$	$\mathcal{Q}$	min card.	$\exists_{i=1}^{n} y_i (\bigwedge_i (R(x,y_i) \wedge C(y_i) \wedge \bigwedge_{j=1}^{n} y_i \neq y_j))$
$\{a\}$	$\mathcal{O}$	nominal	$x = a$
$R_1 \sqsubseteq R_2$	$\mathcal{H}$	role incl.	$R_1(x,y) \rightarrow R_2(x,y)$
$R_1 \circ \cdots \circ R_n \sqsubseteq R$	$\mathcal{R}$	role chain	$(\bigwedge_{i=1}^{n} R_i(x_i, x_{i+1})) \rightarrow R(x_1, x_{n+1}))$
$R_1^- \sqsubseteq R_2$	$\mathcal{I}$	inverse	$R_1(y,x) \rightarrow R_2(x,y)$

Table 2. More description logic constructs (left column), where R_i and R are roles, C is a class, a is an individual, and n is a non-negative integer. The second column gives a letter which is used to identify the construct in the commonly used description logic naming convention. In addition, the letter $\mathcal{S}$ is used as an abbreviation for $\mathcal{ALCH}$ with transitivity axioms of the form $R \circ R \sqsubseteq R$. The letter $\mathcal{N}$ is the restriction of $\mathcal{Q}$ to the case where $C = \top$. $\mathcal{N}$ and $\mathcal{Q}$ are also called *number restrictions.* The last column gives a translation into first-order predicate logic. Role chains are also sometimes called *complex role inclusion axioms.*

Further reasoning tasks that are commonly not of the same computational complexity can be obtained as variations of the previous ones. *Classification* requires to compute subsumptions between all concept names in the knowledge base. *Instance retrieval* focuses on the finding of instances of a given concept, while in the *realization problem*, we are searching for the most specific concept C such that $\mathcal{K} \models C(a)$ for a given individual a. Other non-standard reasoning tasks are also listed as follows because of their potential interest for applications. Among them are *least common subsumer* [Baader *et al.*, 1999b; Küsters and Molitor, 2001], *matching* [Baader *et al.*, 1999a; Baader and Küsters, 2000] and *approximation and difference* [Brandt *et al.*, 2002]. Also of increasing importance are *explanations* for entailed information [Horridge *et al.*, 2008] also considered under *axiom pinpointing* [Kalyanpur *et al.*, 2005; Meyer *et al.*, 2006; Schlobach *et al.*, 2007; Baader and Peñaloza, 2010] and *conjunctive query answering* [Glimm *et al.*, 2008; Eiter *et al.*, 2009; Calvanese *et al.*, 2013a].

Additional constructs have been introduced for extending $\mathcal{ALC}$ while retaining decidability. We give some of the most important ones in Table 2. In some cases, additional global syntactic restrictions have to be enforced to retain decidability.

Description logics which contain $\mathcal{ALC}$ are very expressive but of high computational complexities (see Section 4). Description logics of comparatively low computational complexity (e.g., PTime) have also been introduced more recently, e.g. the logic $\mathcal{EL}^{++}$, which essentially supports only class conjunction ($\sqcap$), the top concept $\top$, existential quantification ($\exists R.C$), nominals, and role chains [Baader *et al.*, 2005].

An example for a description logic knowledge base is given in Figure 1. It describes the formal definition of a so-called ontology design pattern [Gangemi, 2005] for the notion of *trajectory.* The key idea behind the pattern is that a

$$\begin{aligned}\mathit{Fix} \sqsubseteq \exists \mathit{atTime}.\mathit{TemporalThing} \sqcap \exists \mathit{hasLocation}.\mathit{Position}\\ \sqcap \exists \mathit{hasFix}^{-}.\mathit{SemanticTrajectory}\end{aligned} \tag{1}$$

$$\mathit{Segment} \sqsubseteq \exists \mathit{startsFrom}.\mathit{Fix} \sqcap \exists \mathit{endsAt}.\mathit{Fix} \tag{2}$$

$$\top \sqsubseteq {\leq}1\,\mathit{startsFrom}.\top \tag{3}$$

$$\top \sqsubseteq {\leq}1\,\mathit{endsAt}.\top \tag{4}$$

$$\mathit{Segment} \sqsubseteq \exists \mathit{hasSegment}^{-}.\mathit{SemanticTrajectory} \tag{5}$$

$$\mathit{startsFrom}^{-} \circ \mathit{endsAt} \sqsubseteq \mathit{hasNext} \tag{6}$$

$$\mathit{hasNext} \sqsubseteq \mathit{hasSuccessor} \tag{7}$$

$$\mathit{hasSuccessor} \circ \mathit{hasSuccessor} \sqsubseteq \mathit{hasSuccessor} \tag{8}$$

$$\mathit{hasNext}^{-} \sqsubseteq \mathit{hasPrevious} \tag{9}$$

$$\mathit{hasSuccessor}^{-} \sqsubseteq \mathit{hasPredecessor} \tag{10}$$

$$\mathit{Fix} \sqcap \neg\exists \mathit{endsAt}.\mathit{Segment} \sqsubseteq \mathit{StartingFix} \tag{11}$$

$$\mathit{Fix} \sqcap \neg\exists \mathit{startsFrom}.\mathit{Segment} \sqsubseteq \mathit{EndingFix} \tag{12}$$

$$\mathit{Segment} \sqcap \exists \mathit{startsFrom}.\mathit{StartingFix} \sqsubseteq \mathit{StartingSegment} \tag{13}$$

$$\mathit{Segment} \sqcap \exists \mathit{endsAt}.\mathit{EndingFix} \sqsubseteq \mathit{EndingSegment} \tag{14}$$

$$\mathit{SemanticTrajectory} \sqsubseteq \exists \mathit{hasSegment}.\mathit{Segment} \tag{15}$$

$$\mathit{hasSegment} \circ \mathit{startsFrom} \sqsubseteq \mathit{hasFix} \tag{16}$$

$$\mathit{hasSegment} \circ \mathit{endsAt} \sqsubseteq \mathit{hasFix} \tag{17}$$

$$\exists \mathit{hasSegment}.\mathit{Segment} \sqsubseteq \mathit{SemanticTrajectory} \tag{18}$$

$$\exists \mathit{hasSegment}^{-}.\mathit{SemanticTrajectory} \sqsubseteq \mathit{Segment} \tag{19}$$

$$\exists \mathit{hasFix}.\mathit{Segment} \sqsubseteq \mathit{SemanticTrajectory} \tag{20}$$

$$\exists \mathit{hasFix}^{-}.\mathit{SemanticTrajectory} \sqsubseteq \mathit{Fix} \tag{21}$$

Figure 1. Example of a $\mathcal{SRIN}$ knowledge base. It encodes a so-called *ontology design pattern* for the notion of *trajectory*. The example is taken from [Hu *et al.*, 2013].

trajectory consists of a sequence of segments, each of which has a start point and an end point—these points (together with temporal information) are called *fixes* of the segment. Axiom (1) indeed states that each fix has a location and carries temporal information (both of which are not further specified in this pattern). Furthermore, a fix is always a fix of some trajectory. Axiom (2) states that each segment starts from a fix and ends at a fix. The cardinality statements in axioms (3) and (4) state then that these two roles are *functional* in the sense that they represent binary predicates which are in fact functions. Axiom (5) states that every segment is indeed a segment of some trajectory. Axiom (6) uses role chains to ensure that the role *hasNext* connects each fix in a trajectory directly to the next fix in the trajectory. Axioms (11) and (12) identify the first and last fix of a trajectory, while (13) and (14) identify the first and last segment of a trajectory. The remaining axioms further declare relationships between the concepts and roles, please refer to [Hu *et al.*, 2013] for further information about this pattern and the design rationales underlying it. The example can be expressed in the description logic $\mathcal{SRIN}$: Axioms (2), (11) to (15), (18) and (20) are in $\mathcal{ALC}$, role hierarchies ($\mathcal{H}$) are used, e.g., in (7) and also transitivity is used in (8) – $\mathcal{S}$ stands for $\mathcal{ALCH}$ plus transitivity. Role chains ($\mathcal{R}$) are used, e.g., in (17), as well as cardinalities ($\mathcal{N}$) for the functionality statements in (3) and (4), and several occurrences of inverse roles ($\mathcal{I}$).

4 ALGORITHMIC ASPECTS

In the following, we discuss algorithmic advances, the systems in which these are applied, and point out important results in computational complexity [Papadimitriou, 1994], thus drawing an arc from the first still limited DL systems in the early 1990s to the systems of today ranging from highly expressive general purpose DL reasoners to specialized highly efficient reasoners tailored to particular DLs.

In general, many of the first DL systems employ so-called structural subsumption algorithms, in which two descriptions are normalized and then their structure is compared recursively [Nebel, 1990a; Borgida and Patel-Schneider, 1994]. These algorithms are in general efficient (polynomial) for a restricted language but incomplete for more expressive DLs in the sense that not all possible inferences can be derived, and different systems adopt different positions within that scale. Namely, Classic [Brachman *et al.*, 1991] permits only a limited set of constructors such that the computation is efficient and complete, while other approaches, such as Loom [MacGregor and Bates, 1987; MacGregor, 1991] and Back [Nebel and von Luck, 1988; Peltason, 1991], are incomplete but allow for a more expressive language. Further investigations revealed that the source of incompleteness in such systems are certain combinations of constructors in the language and that a slight increase in the expressiveness could turn reasoning intractable [Brachman and Levesque, 1985; Nebel, 1990b]. Additionally, all these systems employ rule-based and/or closed-word reasoning services (mainly on the ABox) which adds further expressiveness to the system, but causes problems since it deviates from the formal semantics due to the additions being rather ad-hoc.

Trying to overcome the limitations of these early DL systems led to the development of sound and complete algorithms for more expressive DLs and subsequently to new systems, such as Kris [Baader and Hollunder, 1991] and CRACK [Bresciani *et al.*, 1995], that were less efficient but expressive and complete. The basic idea behind these new tableau-based algorithms [Schmidt-Schauß and Smolka, 1991; Donini *et al.*, 1991; Hollunder *et al.*, 1990] is trying to find a proof for the unsatisfiability of a concept in a constructive way. If the proof fails, then a canonical model representing a counterexample is obtained. Other reasoning tasks can be achieved by reducing them into (un)satisfiability of a concept, which is always possible for the languages based on $\mathcal{ALC}$ in such systems. Initially, the high worst-case complexity (ExpTime in general for $\mathcal{ALC}$ alone) was considered problematic [Buchheit *et al.*, 1993], but empirical analysis revealed that the combinations of constructors leading to this high complexity are rarely occurring [Nebel, 1990b] and with some optimizations, the performance of a DL system could be considerably improved on average [Baader *et al.*, 1992]. Due to their generality, these new systems also turned out to be useful for comparing and benchmarking other systems [Baader *et al.*, 1992; Heinsohn *et al.*, 1992].

In general, tableau-based algorithms became the dominating approach in DL research for a number of reasons. Namely, the approach is rather flexible, i.e., a variety of languages can be covered by simply adapting the considered tableau expansion rules [Hollunder *et al.*, 1990], but also, if necessary, adopting more advanced mechanisms to ensure termination [Baader, 1991; Buchheit *et al.*, 1993]. It also turned out that, for several DL languages, the worst-case complexity of the algorithm is not worse than the complexity of deciding satisfiability for the logic [Hollunder *et al.*, 1990], making tableaux also a widely used tool in complexity analysis.

At the same time of the appearance of these first tableau-based systems, an alternative for devising algorithms and complexity analysis was introduced by establishing relations to other logical formalisms. For example, it can easily be seen from the translation of DLs to first-order predicate logic in Section 3, that $\mathcal{ALC}$ falls within $\mathcal{L}^2$ [Borgida, 1996], the two-variable fragment of first-order predicate logic, whose decidability was already shown in [Mortimer, 1975]. Not all of the additional constructors shown in Table 2 can be expressed in $\mathcal{L}^2$, but number restrictions can be expressed in $\mathcal{C}^2$, i.e., $\mathcal{L}^2$ extended by counting quantifiers, that is also decidable [Grädel *et al.*, 1997; Pacholski *et al.*, 1997]. However, algorithms building on this correspondence are in general not optimal, i.e., of higher complexity than necessary.

This differs for the relation between multi-modal logic and DLs [Schild, 1991] essentially obtained by viewing $\exists R$ and $\forall R$ as modalities. In fact, $\mathcal{ALC}$ is a variant of the propositional multi-modal logic **K**, and $\mathcal{ALC}$ with transitive closure of roles [Baader, 1991] matches Propositional Dynamic Logic (PDL). This not only yielded the precise complexity of so-called $\mathcal{ALC}_{trans}$ (ExpTime-complete [Fischer and Ladner, 1979]), but was also used in subsequent years to transfer known decidability results from modal logics to DLs [Schild, 1994; De Giacomo and Lenzerini, 1994a; De Giacomo and Lenzerini, 1994b; De Giacomo and Lenzerini, 1996]. Additionally, there exists a strong similarity between algorithms for deciding satisfiability in PDL and the tableau-based algorithms in DLs.

As requested by applications and driven by the just mentioned correspondence and results on tableau-based algorithms for more expressive DLs [Horrocks and Sattler, 1999; Horrocks *et al.*, 2000], the next generation of tableau-based DL systems emerged at the end of the 1990s, namely FaCT [Horrocks, 1998], Race [Haarslev and Möller, 1999], and DLP [Patel-Schneider, 1999]. These reasoners employ considerably more expressive DL languages than before, but the use of sophisticated optimization techniques [Horrocks, 2007] ensures that these reasoners are usable in practice. Continuous further improvements, incorporating and optimizing more and more expressive features, in particular driven by the W3C standard OWL for the Semantic Web, led to highly expressive general purpose DL reasoners based on tableau algorithms incorporating DL languages whose worst-case is N2ExpTime-complete, such as FACT++ [Tsarkov and Horrocks, 2006], Pellet [Sirin *et al.*, 2007], RACER [Haarslev *et al.*, 2012] or Konclude [Steigmiller *et al.*, 2013].

In addition, more recently a number of approaches have been developed that explore alternative algorithms commonly focusing on a restricted language and aiming at more efficient reasoning. Among them is KAON2 [Motik and Sattler, 2006], which is based on ordered resolution as a means of translating $\mathcal{SHIQ}$ DL knowledge bases into disjunctive Datalog. Datalog-based reasoning has also been applied to more restricted DL languages, e.g., for DLP [Grosof *et al.*, 2003], the Horn fragment of DLs, and for $\mathcal{EL}^{++}$ [Krötzsch, 2010]. HermiT [Motik *et al.*, 2009] builds on a combination of hypertableau and hyperresolution for $\mathcal{SHOIQ}^{+}$. Another approach is based on type elimination [Rudolph *et al.*, 2008a; Rudolph *et al.*, 2008b; Rudolph *et al.*, 2012] for $\mathcal{SHIQ}b_s$ extended with DL-safe rules by basically transforming the TBox into ordered binary decision diagrams and then to disjunctive datalog. A further line of investigation follows so-called consequence-based approaches, such as CEL [Baader *et al.*, 2006] for $\mathcal{EL}^{++}$, CB [Kazakov, 2009] for Horn-$\mathcal{SHIQ}$, ConDOR [Simancik *et al.*, 2011] for $\mathcal{ALCH}$, and ELK [Kazakov *et al.*, 2011] again for $\mathcal{EL}^{++}$, that classify the entire ontology in a bottom-up-like fashion achieving considerable better performance than general tableau-based algorithms.

It can be expected, that further research and new requirements from applications will push the limits of current DL systems even further. The ORE workshop [Gonçalves *et al.*, 2013] may be a good indicator for novel tendencies of DL systems, including currently among others the idea of modular reasoners, as well as reasoners for mobile devices with potentially limited resources.

5 RECENT DEVELOPMENTS

We briefly discuss some of the recent research developments regarding Description Logics, and give some key pointers to the literature. Our list is by no means exhaustive; an excellent way to understand the state of the art is to peruse the proceedings of the annual Description Logic Workshop[3] and to follow central Semantic Web outlets, such as the International Semantic Web Conference (ISWC), the Journal of Web Semantics (Elsevier), or the Semantic Web journal (IOS Press).

One of the major trends which started in the mid-2000s was to look at tractable (i.e., polynomial time complexity) Description Logics. Obvious important can-

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[3] See http://dl.kr.org/workshops/.

didates are certain Horn fragments of Description Logics [Grosof *et al.*, 2003; Krötzsch *et al.*, 2008; Krötzsch *et al.*, 2013], but a major stepping stone was the discovery of $\mathcal{EL}^{++}$ [Baader *et al.*, 2005], which does allow tractable standard reasoning tasks, essentially, by excluding the constructors $\neg$, $\sqcup$, and $\forall$, and its application in the life sciences [Baader *et al.*, 2006]. At the same time, the DL-Lite family of Description Logics emerged [Calvanese *et al.*, 2007], that focuses on answering queries, basically by translating a conjunctive query by means of the TBox into an SQL query which can be processed using data base technology. This has gained further momentum recently with Ontology-based Data Access (ObDA) [Calvanese *et al.*, 2011; Calvanese *et al.*, 2013b; Kharlamov *et al.*, 2013], i.e., utilizing an ontology to facilitate data access by providing views and queries solely based on the language of the ontology. The importance of such tractable languages is further emphasized by the fact that they were included in the latest revision of the Web Ontology Language (OWL) standard by the World Wide Web Consortium (W3C) [Motik *et al.*, 2012; Hitzler *et al.*, 2012].

Rather classical KRR topics make their reappearance in the context of Description Logics, usually driven by the need to enhance expressivity. These include, e.g., fuzzy and probabilistic logics [Straccia, 2001; Bobillo and Straccia, 2009; Lukasiewicz and Straccia, 2009; Borgwardt and Peñaloza, 2012; Borgwardt and Peñaloza, 2013; Klinov and Parsia, 2013], temporal logics [Lutz, 2001; Sturm and Wolter, 2002; Artale *et al.*, 2013], and inconsistency handling, either through bugfixing [Huang *et al.*, 2005; Schlobach *et al.*, 2007] or through paraconsistency [Maier *et al.*, 2013]. Novel is the emphasis on decidability and on complexity issues. In particular the latter serve as a type of a-priori assessment of efficient implementability, although typical complexities are very high (see Section 4).

Important are also the relationships to other established reasoning paradigms, in particular the relation to rule-based approaches, see, e.g., [Grosof *et al.*, 2003; Horrocks *et al.*, 2004; Horrocks *et al.*, 2005; Krötzsch *et al.*, 2013; Krötzsch *et al.*, 2008; Krötzsch *et al.*, 2011; Krisnadhi *et al.*, 2011] and also the chapter on *Logics for the Semantic Web* in this volume. It was also argued very early that aspects of the closed world assumption would be required for some application contexts (see, e.g., [Grimm and Hitzler, 2008]), and so non-monotonic extensions of description logics have been created, mostly based on established approaches in the KRR field [Baader and Hollunder, 1995; Donini *et al.*, 1998; Donini *et al.*, 2002; Bonatti *et al.*, 2009; Sengupta *et al.*, 2011], and of course this has led to a combined study of rules and non-monotonicity in relation to DLs, see, e.g., [Eiter *et al.*, 2008; Motik and Rosati, 2010; Knorr *et al.*, 2011; Krisnadhi *et al.*, 2011; Knorr *et al.*, 2012] and the references contained therein.

Concrete domains [Baader and Hanschke, 1991], i.e., the enhanced use of data types, are also considered important for modeling and have drawn renewed attention recently [Lutz, 2004; Lutz *et al.*, 2005; Lutz and Milicic, 2007]. Other investigations are driven by Semantic Web-related use cases, in the wake of the adoption of Description Logics for the W3C Web Ontology Language OWL [McGuinness and van Harmelen, 2004; Hitzler *et al.*, 2012], e.g., distributed knowledge bases

[Borgida and Serafini, 2003], justifications for reasoning results [Horridge *et al.*, 2008; Horridge *et al.*, 2013], or enhancing efficiency by massive parallelization [Schlicht and Stuckenschmidt, 2010; Mutharaju *et al.*, 2013].

6 CONCLUSIONS

We have introduced Description Logics and described their historic roots. We also discussed algorithmic aspects from a historic perspective and considered recent research developments.

Description Logics can be traced back to network-based structures and frames. Once they became established, their development was distinguished from previous approaches to KRR by a focus on complexity and decidability. In the wake of the Semantic Web [Hitzler *et al.*, 2010], and in particular due to their adoption as one of the main Semantic Web standards [McGuinness and van Harmelen, 2004; Hitzler *et al.*, 2012]. Research on theoretical and practical aspects of Description Logics is still going strong, and its development in the near and intermediate future will likely depend on further developments related to Semantic Web technologies.[4]

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[4] See the chapter on Logics for the Semantic Web, in this volume.

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